9. THE INEFFABLE

1. Proofs Of The Existence of God

We’ve finished the mathematical content of this book. This final chapter consists of some philosophical/theological musings that arise in some minds as a result of encountering those parts of mathematics that deal with the edge of the rational universe. If you have no interest in the fundamental questions of life then it’s best that you skip this chapter.

Now I wish to make it clear that my purpose in writing these notes is to communicate what I see as the nature of mathematics, not to talk about religion. I once received an email from an angry reader who believed that this chapter was the “pill” and the previous ones were the “sugar coating” and that my whole aim was to sneak religion in under the radar.

If you are one of those who get angry at the mere mention of anything religious then you’d better not read on. But let me emphasise that I am not here talking about the ineffability of God, but the ineffability of mathematics. I present some “proofs” of the existence of God, merely as a vehicle for discussing bad, or erroneous logic. But, by the same token, the fact that I expose such faulty reasoning does not mean that I am arguing against the existence of God.

I lay no claim to any professional expertise in either philosophy or theology. But I can’t help going beyond the mathematics of the ineffable to the ineffable itself.

The word “ineffable” means “inexpressible in words”. It’s a word that not only appears in hymns, describing God, but also in a large number of nineteenth century novels. We don’t use the word today, yet there’s as much interest in the transcendental as ever.

There’s a fundamental contradiction in the desire to discuss the ineffable – to say something meaningful about something that can’t be expressed in words. But by a little distortion of the meaning we can think of the ineffable as that which transcends logic.

Can one prove that God exists? There have been many attempts over the centuries. A very simplistic argument, at least in the Christian tradition, runs as follows. The bible says that God exists. The bible says that everything in the bible is the word of God and so must be true. Therefore God exists. Put more simply it says “GOD EXISTS AND THIS STATEMENT IS TRUE”.

One need not spend too much time in refuting this feeble argument. Just one word is needed – self-referentiality. But apart from that it would be logically consistent to believe that the statement is false. It would have been far better if the Bible had made just two claims:

| verse 1: God exists. | verse 2: Everything in the Bible is false. |

If verse 2 is true then both verse 1 and verse 2 are false. But this leads to a contradiction. Oh well then, verse 2 is false. So it is false that everything in this Bible is false. So something must be true. It can’t be verse 2 because we’re assuming that it’s false. It must therefore be verse 1 that’s true.
This might seem momentarily convincing until we realise that any statement could have taken the place of “God exists” – even “God does not exist”. The problem lies in the fact that verse 2 is “self-referential” – it refers to itself.

One must refrain from considering any sentence that refers to its own truth. Such self-referential statements are meaningless and meaningless statements used in a logical argument can lead to paradoxes such as the above.

2. The Third Floor Brothel

The human mind is a wonderful thing – especially the mind of a mathematician. The phenomenon of the subconscious mind working on problems while the conscious mind is otherwise occupied may exist in other walks of life, but being a mathematician I am only conscious of it in relation to mathematical problem-solving.

The history of mathematics contains numerous examples of a mathematician striving unsuccessfully to solve a problem, and them, while carrying out some unrelated activity, the solution pops into his brain. I’m not sure what Archimedes was thinking in his bath, but it may not have been the principle that came in that famous “Aha” moment.

Though a lesser mathematician than Archimedes, or any of the other “men of mathematics” I can report this happening to me on many occasions. In fact my standard advice to mathematics students, preparing to sit for an exam, is to read the whole paper first and to think about each question for a few minutes before putting pen to paper. I have had many students report back that on thinking about question 5 they had no idea how to tackle it, but an hour later when they had finished question 4, question 5 seemed to have fallen into place, even though they hadn’t been actively thinking about it in the meantime.

A related phenomenon is dreaming mathematics. This has happened to me on more than one occasion. I once woke up after having thought of a wonderfully clever examination question which I actually used. I hasten to admit that there have probably been more occasions when I have woken up having solved some big problem, only to discover that the logic was faulty.

But let me tell you of one occasion when I woke up with a story in my mind where the fact that the logic was subtly flawed was the whole point.

In the dream I was living opposite a brothel, operating out of the top floor apartment in a three-story block. There was a red neon sign that announced this fact in a subtle way. (The fact that some years earlier my daughter happened to live opposite such a brothel may have contributed to this detail.)

In the dream I had complained to the local council about an illegal brothel and one night the mayor and two aldermen came around to investigate. (It seems a bit far-fetched that they would come themselves rather than send round a council employee but, as I said, I have changed very few details.)

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The red light wasn’t there and they were quizzing me about it. I said that it had been there last night. They discussed the situation among themselves, and it appeared that the fact that was a brothel seemed to be irrelevant. It was a commercial activity in a building zoned as residential. Even that didn’t seem to pose an insurmountable problem, they said. Maybe the building should be rezoned as commercial.

We all went across the road. The bottom apartment was rented by a couple of young, single men, and as we were standing outside one of the men came out onto the balcony to have a smoke in the cool night air.

The mayor asked him whether he thought the block should be zoned commercial, or remain residential. Now the man not only knew about the brothel – he was one of their best customers. But he must have thought to himself that if it was rezoned the landlord would

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want a commercial tenant for his own flat at a much higher rent. So he said emphatically that it should remain residential.

By this stage the tenants on the higher floors had come out onto their balconies. In the middle floor apartment there lived a young family. Should the block remain residential? Most definitely yes! They didn’t want a brothel, legal or illegal operating above them.

“What about you?” one of the aldermen called out to the two young ladies on top floor balcony.

“We think it should be rezoned commercial”, one of them called down.

“That’s two votes out of three for the status quo”, said the other alderman.

“Let’s enquire further”, urged the mayor. He called up to the young mother on the middle level, “do you think the lady above you is right?” he called out.

“No, she’s definitely not correct”, replied the indignant mother.

“What about you”, the mayor said to the young man on the bottom floor. “Do you think that lady on the floor above you is right?”

“Yes, she is”, he answered.

“What about you”, the mayor yelled to the lady on the top floor. “Do you think this young man is right?”

Now the ladies on the top hadn’t heard what the man had said, but they knew what he would say. “Yes, he’s most correct.” Of course they thought that he would be in favour of the block being rezoned.

“Well that’s good”, said the first alderman. “if the top floor ladies are correct then so is the bottom floor man. And he said that the lady in the middle flat was right. So they are unanimous – they’re all correct.”

“But,” said the second alderman, “that middle lady said that the top floor ladies were wrong – that’s a contradiction”.

“Well, all that means”, said the mayor who knew a little bit of logic, “is that the top floor ladies can’t be correct.”

“But”, I said, “she said that the bottom-floor man was right, so he must be wrong. And he said that the middle lady was right, so she must be wrong – all three are wrong!”

The second alderman rubbed his chin and said, “but if the middle lady is wrong about the top-floor lady being wrong then the top floor lady must be right!”

“Oh dear, that’s another contradiction”, said the mayor. “We’ll never get this sorted out!”

Are you confused? Let me try to make the paradox clearer by taking out the irrelevant detail.

**Bottom Floor:** Middle floor is right.
**Middle Floor:** Top floor is wrong.
**Top Floor:** Bottom floor is right.

Let’s abbreviate the bottom, middle and top floor people to B, M and T respectively.

If T is right, then B is right and so M is right and so T is wrong, a contradiction.
If T is wrong then B is wrong and so M is wrong and so T is right, a contradiction.

Now to some extent we could explain this away by observing that they were referring to the original question of the zoning, not to whether the adjacent neighbour was right. But suppose that they gave the above responses to the correctness of their neighbour. We have a logical paradox.
You could explain it away on the grounds that the lady on the top floor didn’t hear what the man at the bottom had said – she was responding to what she thought he would have said. But what if they gave the above responses to the question “is your neighbour right or wrong”?

3. Self-Referential Systems

Let’s make the matter even simpler by considering the following “party game”. Well, it’s not actually a game, but you can carry it out at a party – whether it provokes any interest is another matter.

Get five people to stand around in a circle. Give each of them a piece of paper with a question on it, ensuring that no-one else can see each other’s question. Tell them that each of them has a question which they must answer aloud. As far as each of the five knows they all have different questions. But in fact every question reads as follows:

Will the person on your right will answer his or her question correctly.

Nobody will know what question the person on the right is being asked, so it’s a pure guess, but of course they may look closely at that person to see whether they appear devious, or even a little stupid. But they must make a guess and call out “YES” or “NO”. Once they’ve all called out their answers you reveal that everyone had the same question.

If an even number of people called out “NO”:

Focus on any one of the five and ask them, “did you feel that you were able to choose your answer freely, or did you feel compelled to answer the way you did?” Of course they’ll say that they could easily have chosen the other alternative.

“But that’s not so”, you reply, “logic compelled you to answer the way you did.” You then analyse the situation if they had chosen otherwise by working out the implications of their answer being right or wrong. In either case you will demonstrate a contradiction. “Therefore, you had to choose as you did”.

For example suppose the five people called out answers as follows:

1. YES, 2. NO, 3. NO, 4. YES, 5. YES

with person 1 being the one you are addressing. If he had called out NO instead you would have had the answers being:

1. NO, 2. NO, 3. NO, 4. YES, 5. YES

Now if person 1 was right, person 2 would be wrong, person 3 would be right, person 4 would be wrong, person 5 would be wrong and so person 1 would be wrong. Think about it carefully. This leads to a contradiction.

On the other hand if person 1 was wrong, person 2 would be right, person 3 would be wrong, person 4 would be right and person 5 would be right and so person 1 would be right, again a contradiction.

“To avoid a contradiction you were logically compelled to call out “YES”.

If an odd number of people called out “NO”:

Carry out the above analysis to get a contradiction and ask which of them is guilty of defying logic!

With a chain of this type, an odd number of NOs will lead around the circle and produce a contradiction. It seems clear that self-referentiality must include multiple statements that go round in a loop. We must avoid that sort of self-referentiality. But can we
be sure that by not having any system that loops back on itself we can avoid all logical paradoxes? Might there not be something more subtle that must be avoided?

Consider the following infinite list of statements:

\[
\text{AT LEAST ONE OF THE FOLLOWING STATEMENTS IS FALSE} \\
\text{AT LEAST ONE OF THE FOLLOWING STATEMENTS IS FALSE} \\
\text{AT LEAST ONE OF THE FOLLOWING STATEMENTS IS FALSE} \\
\vdots
\]

At first sight we have repeated the same statement infinitely many times, but look more closely. The phrase “following statements” refers to a different collection each time. So one of these could be true and another false.

Now if any of these statements is false then all the following must be true. (If it isn’t true that one of them is false then they must all be true.) So they must all be true from some point on. But how can that be. If any one of these statements is true there must be a false statement below it!

Here we have a logical paradox with no self-referentiality. No statement, directly or indirectly, refers to itself.

So we have to rule out, not only self-referential systems, but also any infinite chain of statements. Is that all? Can we be sure that there aren’t even more subtle problems with logic? We are getting rather close to the limit of human thought here, so let’s return to the question of whether there is a logical proof of the existence of God.

4. Proof By Definition

I remember, when training for my accreditation as a lay preacher many years ago, that I had to study many of these arguments – mostly with big names like “the ontological argument”. I mostly forget what they were.

One of the ones I do remember went like this. We define God to be a being that is perfect in every way. Now existence is more perfect than non-existence. So if God did not exist this would contradict our definition. Therefore God exists.

The problem with this argument is that it assumes the existence of a being that is perfect in every way but who does not exist. The contradiction comes from assuming simultaneously the existence and the non-existence and has nothing to do with perfection.

We might define “infinity” as “a number that is bigger than every number” and ask the question, “Does this infinity exist?” Well a number that doesn’t exist cannot be bigger than every number. (In fact a non-existent number cannot be bigger than any number.) Therefore infinity must exist. But, of course, such an “infinity” must be bigger than itself, a situation that is clearly untenable.

The explanation for this paradox is that “not existing” is not a property of something. It is the absence of something with a given property. We could say that a non-existent number cannot be even. But nor can it be odd. “n is even” is not true or false of a non-existent number – it is meaningless. In the same way “God is perfect” is not true or false of a non-existent God. It is meaningless.

5. Proof By Design

Another proof that God exists, one that was very popular in Victorian times, is Proof By Design. The world is a complex, finely balanced precision structure. If certain parameters were changed by only a small amount life would not be possible. It could not possibly have arisen by chance. There must have been an intelligent Creator. A watch could not come
about by cogs just throwing themselves together. So the universe must have been created by a Divine Clockmaker.

But then along came Darwin and his Theory of Evolution. Complexity can arise from simple rules. These days Proof by Design is used in the Creationism versus Evolution debate rather than in the Exist/Does Not Exist one.

There’s certainly more strength in Proof by Design than in the artificial word games that we started with. At least there is some substance to the argument, even if it can be refuted by equally plausible counter-arguments. But in no way could it be considered as a proof.

The scientific discoveries of the nineteenth centuries were used to support Proof by Design. But some scientific discoveries of the twentieth centuries have supported a Proof of the Non-Existence of God. The more we discover about the workings of the human brain the more we can explain our thoughts and emotions in biological ways. We have learnt how these non-material entities can be influenced by our biochemistry. There’s a popular belief – more widely accepted by lay people whose scientific training is derived from newspapers and television than by the scientists themselves – that all our thoughts and all our desires are purely the product of biochemical processes. We eat, therefore we think. Electrical impulses stimulate a certain part of our brain and we feel a corresponding emotion.

Such a materialistic explanation of our thoughts clearly precludes the possibility of God. A belief in God is the result of wishful thinking, perhaps a result of a certain God-gene being switched on.

But wait! Do we detect some self-referentiality going on here? “I have concluded, by sound logical reasoning, that all thought is biochemical and depends only on a cocktail of chance events such as the genes I inherited, the food I’ve just eaten – perhaps topped off with a dash of some stray gamma rays from outer space.”

Or perhaps the brains of scientists who make such materialistic claims are somehow immune from the phenomenon they are describing. The world, including our brains, might operate in a purely deterministic way and we might believe that it is so. But clearly such a belief can have no validity and the fact we believe in what is indeed the state of the world must be purely coincidental.

To make any statement about the world implicitly includes the assertion that rational thought is possible and not a purely deterministic biological process. In other words, to claim that “rational thought is impossible” is a contradiction.

Of course this does not prove that rational thought is possible. Perhaps thinking logically is just an illusion after all. But to any one with a materialistic belief there are just two possibilities. Their belief is either meaningless or false. At least for the person who believes in something beyond the material world there are the two possibilities: their belief is either meaningless or true. I know which I’d rather! But believing that there is something beyond this material world falls a long way short of believing in God, so we haven’t got very far.

6. Faith As A Set Of Axioms

What we’ve learnt from the earlier chapters is that there is no such thing as a proof without axioms and while we may assert that our axioms are self-evident this may be far from the case.

Christians, and probably followers of other religious traditions, are often ridiculed because of their need to accept certain things without proof. Indeed we usually refer to a religious system as a “faith”. Surely an intelligent person will only believe what they can prove.
But in the absence of axioms – a collection of starting assumptions – one can prove nothing. And the axioms, by their very nature, don’t require proof. At its simplest, a religious faith is the acceptance of a set of axioms on the basis of which further beliefs, and more importantly, actions are based. How one arrives at such a set of axioms involves aspects of the mind that go far beyond the purely logical. I’m not qualified to discuss this wider world of reasoning. All I can say is that, as a mathematician, I’m fully aware that truth transcends logic in the strict mathematical sense.

### 7. Agnosticism

We haven’t actually proved that one cannot prove the existence of God, but it seems quite likely. Surely this is a good argument for agnosticism. I mentioned in an earlier chapter that the existence of an infinite number between $\aleph_0$ and $2^{\aleph_0}$ is undecidable. A proof exists that it is impossible to prove that such a number exists and another proof exists that it is impossible to prove that no such number exists. I suggested there that, since we’re logically free to accept or deny the Continuum Hypothesis (that there is no such number), it makes good sense to accept it on the grounds that we’re never going to find one, so for all practical purposes they don’t exist.

Someone might postulate the existence of a universe, parallel to ours, that has no interaction with our own. Clearly its existence is undecidable, but it would be bizarre to waste any effort in believing it. Can’t the same practical wisdom be extended to the existence of God?

But there is a difference. An intermediate number between the above two infinities could only interact on the logical part of my brain. Therefore, since its existence is undecidable, I lose nothing by denying it. It actually simplifies things.

The parallel world that I referred to can’t interact with me in any way. A God, if there is one, might interact with me in ways other than through logic. I guess it all boils down to whether you believe that the logical world is all there is.

The agnostic recognises that one can neither prove nor disprove the existence of God and so sits on the fence. The advantage of this position is never being proved wrong. The disadvantage is to be doomed forever to sit in such an uncomfortable place. But it’s remarkable that the agnostic rarely extends his agnosticism into other areas of life.

If I can’t prove that the woman I’m in love with is the most suitable person for me, I won’t marry her. If I can’t prove that the car I’m thinking of buying is the best value for money I’ll put off buying a car in case a better one comes on the market.

The majority of agnostics are not consistent about their agnosticism and what they mean by saying “I’m an agnostic” is “I can’t be bothered”. And that’s an honourable position to be in. I’m agnostic on many questions such as those about football, for example.

But there are a few agnostics who carry their agnosticism to extremes in their everyday lives. In the novel *The Age of Reason* by Jean-Paul Sartre the main character is obsessed by his desire to retain his free will. As a result he makes no commitments. “If I marry her I’ll remove, or at least reduce, my chance of marrying someone else.” Every decision involves a certain reduction in freedom. Better not make any decisions. So the person who’s so determined to maximise his free will locks himself into a prison of indecision. He ends up with less freedom. Free will is a currency that must be spent or it becomes worthless.

Of course there may be no such thing as free will at all. Perhaps all our decisions are completely determined, just as our thoughts and emotions are. In that case there are no true statements and no false ones either. For some that’s how they would like the world to be. Certainly my cat thinks so!