

1. Use Newton's method to estimate the following, correct to 3 decimal places (that is, it is sufficient to continue iterating Newton's method until you have the first 3 decimal places the same in x_n and x_{n+1} , for some n):

(a) $\sqrt{5}$ with initial estimate $x_0 = 2$, (b) $\sqrt{7}$ with initial estimate $x_0 = -3$, (c) $\sqrt[3]{2}$.

2. Sketch the curves $y = \tan x$ and $y = e^x$.

(a) How many solutions are there to the equation $\tan x = e^x$?

(b) What initial guess x_0 will cause Newton's method to fail?

(c) Make an initial guess x_0 that is likely to lead to a solution between $-\frac{1}{2}\pi$ and $\frac{1}{2}\pi$.

(d) Use Newton's method with your choice of x_0 in part (c) to approximate this solution (correct to 3 decimal places).

3. Consider the collection \mathcal{S} of all 7-digit numbers where each digit is non-zero.

(a) How many numbers are there in the collection \mathcal{S} ?

(b) How many numbers in \mathcal{S} have distinct digits?

(c) How many numbers in \mathcal{S} have 1 as the first digit?

(d) How many numbers in \mathcal{S} have distinct digits as well as 2 as first digit and 4 as last digit?

4. In how many ways can one choose 12 people from 20 people and seat them: (i) in a row from left to right? (ii) in a circle? (iii) in a square with 3 on each side? (iv) in a triangle with 4 on each side? (v) in two rows of 6 facing each other? (For these questions, it is only the relative seating positions that matter, not each person's absolute position.)

5. Determine the number of ways of arranging the letters of:

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and leave your answer in terms of factorials.

6. We wish to choose 10 cards from a usual deck of 52 playing cards.

(a) In how many ways can we achieve this?

(b) In how many ways can we achieve this if we are required to choose (i) all cards from the same suit; or (ii) exactly 3 Aces and exactly 3 Kings?

(c) In how many ways can we achieve this if we are required to choose cards of different values (assuming that the 13 cards in each suit are of different values)?

7. Find two positive numbers whose sum is 100 and the sum of whose squares is a minimum.

8. A box with a square base and open top must have a volume of $32,000 \text{ cm}^3$. Find the dimensions of the box that minimize the amount of material used.

9. Find the point on the line $y = 2x - 3$ that is closest to the origin.
10. A piece of wire 10m long is cut into two pieces. One piece is bent into a square, the other is bent into an equilateral triangle.
How should the wire be cut so that the total area enclosed is a maximum?
11. A metal merchant buys some scrap sheet metal in the shape of a semicircle of 1 metre radius. This is an awkward shape to transport and store, so he decides to trim it to a rectangular shape by making a straight cut parallel to the straight side and two straight cuts at right angles to the straight side. Find the dimensions of the largest rectangle he can obtain this way.
12. If we roll two dice, what is the probability that the sum of the numbers is a multiple of 3?
13. Use the binomial expansion to find:
- the coefficient of x^6 in the expansion of $(x + 2)^9$;
 - the coefficient of x^5 in the expansion of $(3x - 4)^{11}$.
14. (a) Let $z = \frac{1}{2}(1 + i)$. Find: (i) \bar{z} , (ii) z^2 , (iii) $|z|$, (iv) $\arg z$, (v) z^{-1} .
(b) Plot z , \bar{z} , z^2 , and z^{-1} on an Argand diagram.
(c) For $z = 3 - 4i$, (i) find \bar{z} , $|z|$ and z^{-1} ; and (ii) verify that $z^{-1} = \frac{\bar{z}}{|z|^2}$.
15. Given complex numbers $z = -2 + 3i$ and $w = 5 - 4i$ evaluate each of:
- \bar{z}
 - zw
 - $z+w$
 - $5w-3z$
 - $\frac{w}{z}$
 - $|w|$
 - $z^2+w\bar{w}$
 - $\frac{zw}{z+w}$
 - $\frac{1}{z-w}$
 - $z\bar{w} + \bar{z}w$
 - $z^2 + w^2$
 - $|z|^2 + |w|^2$.